Fill Ups of Functions

Q. 1. The values of f(x) $= 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ **lie in the interval**

Ans. $\left[0, \frac{3}{\sqrt{2}}\right]$

Solution. For the given function to be defined

$$\frac{\pi^2}{16} - x^2 \geq 0 \Rightarrow -\pi/4 \leq x \leq \pi/4$$

$$\therefore D_f = [-\pi/4, \pi/4]$$

Now, for $x \in [-\pi/4, \pi/4]$, $\sqrt{\pi^2/16 - x^2} \in [0, \pi/4]$

And sine function increases on [0, π/4]

$$\therefore 0 = \sin 0 \leq \sin\sqrt{\frac{\pi^2}{16} - x^2} \leq \sin \pi/4 = 1/\sqrt{2}$$

$$\Rightarrow 0 \leq 3\sin\sqrt{\frac{\pi^2}{16} - x^2} \leq 3/\sqrt{2}$$

$$\therefore f(x) = [0, 3/\sqrt{2}]$$

Q. 2. For the function

$$f(x) = \frac{x}{1 + e^{1/x}}, \quad x \neq 0$$
$$= 0 \quad , \quad x = 0$$

The derivative from the right, f '(0+) = ..., and the derivative from the left, f '(0–) =...............

Ans. 0, 1

Solution.

$$f(x)=\begin{cases}\dfrac{x}{1+e^{1/x}}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$f'(0^+) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{h}{1+e^{1/h}}}{h} = \lim_{h\to 0}\frac{1}{1+e^{1/h}}$$

$$=\lim_{h\to 0}\frac{e^{-1/h}}{e^{-1/h}+1}=\frac{0}{1}=0$$

$$f'(0^-) = \lim_{h\to 0}\frac{f(0)-f(0-h)}{h}$$

$$=\lim_{h\to 0}\frac{0-\frac{-h}{1+e^{-1/h}}}{h} = \lim_{h\to 0}\frac{1}{1+e^{-1/h}}=1$$

Thus $f'(0^+) = 0$ and $f'(0^-) = 1$

Q. 3. The domain of the function $f(x)=\sin^{-1}(\log_2\frac{x^2}{2})$ is **given by................**

Ans. [–2, –1] ∪ [1, 2]

Solution. To find domain of function $f(x)=\sin^{-1}\left(\log_2\frac{x^2}{2}\right)$

For f (x) to be defined we should have $-1\le\log_2\left(\frac{x^2}{2}\right)\le 1$

NOTE THIS STEP:

$$\Rightarrow\quad 2^{-1}\le\frac{x^2}{2}\le 2^1 \quad\Rightarrow\quad 1\le x^2\le 4$$
$$\Rightarrow -2\le x\le -1 \text{ or } 1\le x\le 2$$
$$\Rightarrow x\in[-2,-1]\cup[1,2]$$

Q. 4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is................ and out of these................ are onto functions.

Ans. n^n, $n^!$

Solution. Set A has n distinct elements. Then to define a function from A to A, we need to associate each element of set A to any one the n elements of set A. So total number of functions from set A to set A is equal to the number of ways of doing n jobs where each job can be done in n ways. The total number such ways is n × n × n ×.... × n (n - times).

Hence the total number of functions from A to A is n^n.

Now for an onto function from A to A, we need to associate each element of A to one and only one element of A. So total number of onto functions from set A to A is equal to number of ways of arranging n elements in range (set A) keeping n elements fixed in domain (set A). n elements can be arranged in n! ways.

Hence, the total number of functions from A to A is n!.

Q. 5. If $f(x) = \sin \ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)$, **then domain of f(x) is .... and its range is ....**

Ans. (-2, 1), [-1, 1]

Solution. The given function is,

$$f(x) = \sin\left[\ell n\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right]$$

For ℓn to be defined $\frac{\sqrt{4-x^2}}{1-x} > 0 \Rightarrow 1-x > 0$

Also $4 - x^2 > 0 \Rightarrow x < 1$ and $-2 < x < 2$

Combining these two inequalities, we get x ∈ (– 2, 1)

∴ Domain of f is (– 2, 1)

Also sin q always lies in [– 1, 1].

∴ Range of f is [– 1, 1]

Q. 6. There are exactly two distinct linear functions, ..., and .... which map [– 1, 1] onto [0, 2].

Ans. x + 1 and - x+ 1

Solution. KEY CONCEPT: Every linear function is either strictly increasing or strictly decreasing. If f (x) = ax + b, Df = [p, q], Rf = [m, n]

Then f (p) = m and f (q) = n, if f (x) is strictly increasing and f (p) = n, f (q) = m, If f (x) is strictly decreasing function.

Let f (x) = ax + b be the linear function which maps [–1, 1] onto [0, 2]. then f (–1) = 0 and f (1) = 2 or f (–1) = 2 and f (1) = 0

Depending upon f (x) is increasing or decreasing respectively.

⇒ – a + b = 0 and a + b = 2 ....(1)

or – a + b = 2 and a + b = 0 ....(2)

Solving (1), we get a = 1, b = 1.

Solving (2), we get a = – 1, b = 1

Thus there are only two functions i.e., x + 1 and – x + 1.

Q. 7. If f is an even function defined on the interval (-5, 5), then four real values of x satisfying the equation $f(x)=f\left(\frac{x+1}{x+2}\right)$

are ...., ..., ..., and ....

Ans. $\frac{-3\pm\sqrt{5}}{2}, \frac{3\pm\sqrt{5}}{2}$

Solution. Given that $f(x)=f\left(\frac{x+1}{x+2}\right)$ and f is an even function

$\therefore f(x)=f(-x)=f\left(\frac{-x+1}{-x+2}\right)$

$\Rightarrow x=\frac{-x+1}{-x+2} \qquad \Rightarrow x^2-3x+1=0 \quad \Rightarrow x=\frac{3\pm\sqrt{5}}{2}$

Also $f(x) = f\left(\frac{x+1}{x+2}\right) = f(-x)$

$\Rightarrow \frac{x+1}{x+2} = -x \Rightarrow x^2 + 3x + 1 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{5}}{2}$

$\therefore$ Four values of x are

$\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}$

Q. 8. If f(x) = sin^2 x + $\sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cos\left(x+\frac{\pi}{3}\right)$ *and* $g\left(\frac{5}{4}\right) = 1$, then (gof) (x) =.................

Ans. 1

Solution.

$f(x) = \sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cos\left(x+\frac{\pi}{3}\right)$

$\Rightarrow f(x) = \sin^2 x + \left[\sin\left(x+\frac{\pi}{3}\right)\right]^2 + \cos x \cos\left(x+\frac{\pi}{3}\right)$

$\Rightarrow f(x) = \sin^2 x + \frac{1}{4}(\sin x + \sqrt{3}\cos x)^2 + \frac{1}{2}\cos x(\cos x - \sqrt{3}\sin x)$

$= \frac{5}{4}(\sin^2 x + \cos^2 x) = \frac{5}{4}$

$\therefore \quad (gof)x = g[f(x)] = g(5/4) = 1$

True False of Functions

Q. 1. If $f(x) = (a - x^n)^{1/n}$ where $a > 0$ and n is a positive integer, then $f[f(x)] = x$.

Ans. T

Solution. $f(x) = (a - x^n)^{1/n}$, $a > 0$, n is + veinteger

$f(f(x)) = f[(a - x^n)^{1/n}] = [a - \{(a - x^n)^{1/n}\}n]^{1/n}$

$= (a - a + xn)1/n = x$

Q. 2. The function $f(x) = \dfrac{x^2+4x+30}{x^2-8x+18}$ **is not one -to -one.**

Ans. T

Solution. KEY CONCEPT : A function is one-one if it is strictly increasing or strictly decreasing, otherwise it is many one.

$$f(x) = \frac{x^2+4x+30}{x^2-8x+18} \Rightarrow \quad f'(x) = \frac{-12[x^2+2x-26]}{(x^2-8x+18)^2}$$

$$\Rightarrow \quad f'(x) = \frac{-12(x-3\sqrt{3}+1)(x+3\sqrt{3}+1)}{(x^2-8x+18)^2}$$

$\Rightarrow$ $f(x)$ increases on $\left(-3\sqrt{3}-1, 3\sqrt{3}-1\right)$ and decreases otherwise.

$\Rightarrow$ $f(x)$ is many one.

Q. 3. If $f_1(x)$ and $f_2(x)$ are defined on domains D_1 and D_2 respectively, then $f_1(x) + f_2(x)$ is defined on $D_1 \cup D_2$.

Ans. F

Solution. We know that sum of any two functions is defined only on the points where both f_1 as well as f_2 are defined that is $f_1 + f_2$ is defined on $D_1 \cap D_2$.
$\therefore$ The given statement is false.

Subjective Questions of Functions

Q. 1. Find the domain and range of the function $f(x)=\frac{x^2}{1+x^2}$. **Is the function one-to-one?**

Solution. Since f (x) is defined and real for all real values of x, therefore domain of f is R.

Also $\frac{x^2}{1+x^2} \geq 0$, for all $x \in R$

and $\frac{x^2}{1+x^2} < 1$ $(\because x^2 < 1 + x^2)$ for all $x \in R$

$\therefore \quad 0 \leq \frac{x^2}{1+x^2} < 1 \Rightarrow 0 \leq f(x) < \Rightarrow$ Range of $f = [0, 1)$

Also since f (1) = f (–1) = ½

∴ f is not one-to-one.

Q. 2. Draw the graph of y = | x |$^{1/2}$ for – 1 $\leq$ x $\leq$ 1.

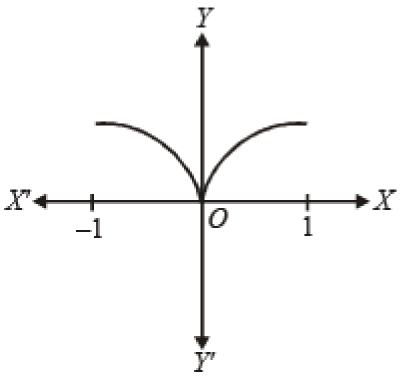
Solution.

$y = |x|^{1/2}, -1 \leq x \leq 1$

$\Rightarrow \quad y = \sqrt{-x}$ if $-1 \leq x \leq 0 = \sqrt{x}$ if $0 \leq x \leq 1$

$\Rightarrow \quad y^2 = -x$ if $-1 \leq x \leq 0$ and $y^2 = x$ if $0 \leq x \leq 1$

[Here y should be taken always + ve, as by definition y is a + ve square root].
Clearly y^2 = – x represents upper half of left handed parabola (upper half as y is + ve) and y^2 = x represents upper half of right handed parabola.
Therefore the resulting graph is as shown below:



Q. 3. If f (x) = $x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$, find f (6).

Solution. f (x) = $x^9 - 6x^8 - 2x^7 + 12x^6 + x^4 - 7x^3 + 6x^2 + x - 3$

Then f (6) = $6^9 - 6 \times 6^8 - 2 \times 6^7 + 12 \times 6^6 + 6^4 - 7 \times 6^3 + 6 \times 6^2 + 6 - 3$

= $6^9 - 6^9 - 2 \times 6^7 + 2 \times 6^7 + 6^4 - 7 \times 6^3 + 6^3 + 6 - 3 = 3$

Q. 4. Consider the following relations in the set of real numbers R.

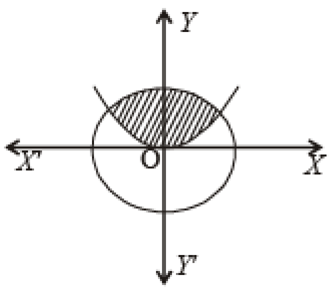
$$R = \{(x, y); x \in R, y \in R, x^2 + y^2 \leq 25\}$$
$$R' = \left\{(x, y) : x \in R, y \in R, y \geq \frac{4}{9}x^2\right\}$$

Find the domain and range of R ∩ R'. Is the relation R ∩ R' a function?

Solution. R = [(x, y); x ∈ R, y ∈ R, x^2 + y2 ≤ 25] which represents all the points inside and on the circle $x^2 + y^2 = 52$, with centre (0, 0) and radius = 5

$$R' = \left\{(x, y) : x \in R, y \in R, y \geq \frac{4}{9}x^2\right\}$$

Which represents all the points inside and on the upward parabola $x^2 \leq \frac{9}{4}y$.



Thus $R \cap R' = R \cap R'$. = The set of all points in shaded region.
For

$$x^2+y^2 \le 25$$
$$\Rightarrow \quad x^2 \le 25-y^2 \qquad \text{....(1)}$$

and $y \ge \frac{4}{9}x^2 \Rightarrow \frac{16x^4}{81} \le y^2 \ \Rightarrow \ -\frac{16x^4}{81} \ge -y^2$

$$\Rightarrow \quad 25-\frac{16x^4}{81} \ge 25-y^2 \qquad \text{....(2)}$$

$\therefore$ Combining (1) and (2) $x^2 \le 25-\frac{16}{81}x^4$

$\Rightarrow$ $16x^4+81x^2-2025 \le 0$

$\therefore$ Domain of $R \cap R' =$

$\{x : x \in R, 16x^4+81x^2-2025 \le 0\}$ and range of $R \cap R'$

$= \{y : y \in R, y \ge \frac{4x^2}{9}, 16x^4+81x^2-2025 \le 0\}$

$R \cap R'$ is not a function because image of an element is not unique , e..g., (0, 1), (0, 2), (0, 3)..... $\in R \cap R'$.

Q. 5. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B.

Solution. As there is an injective maping from A to B, each element of A has unique image in B. Similarly as there is an injective mapping from B to A, each element of B has unique image in A. So we can conclude that each element of A has unique image in B and each element of B has unique image in A or in other words there is one to one mapping from A to B. Thus there is bijective mapping from A to B.

Q.6. Let f be a one-one function with domain {x, y, z} and range {1, 2, 3}. It is given that exactly one of the following statements is true and the remaining two are false f (x) = 1, f (y) ≠ 1, f (z) ≠ 2 determine $f^{-1}(1)$.

Solution. f is one one function,

Df = {x, y, z}; Rf = {1, 2, 3}

Eaxctly one of the following is true :

f (x) = 1, f (y) ≠ 1, f (z) ≠ 2

To determine f –1 (1):

Case I: f (x) = 1 is tr ue.

⇒ f (y) ≠ 1, f (z) ≠ 2 are false.

⇒ f (y) = 1, f (z) = 2 are true.

But f (x) = 1, f (y) = 1 are true, is not possible as f is one to one.

∴ This case is not possible.

Case II: f (y) ≠ 1 is true.

⇒ f (x) = 1 and f (z) ≠ 2 are false

⇒ f (x) ≠1 and f (z) = 2 are true

Thus, f (x) ≠ 1, f (y) ≠ 1, f (z) = 2

⇒ Either f (x) or f (y) = 2. So, f is not one to one

∴ This case is also not possible.

∴ f (z) ≠ 2 is true

∴ f (x) = 1 and f (y) ≠ 1 are false.

⇒ f (x) ≠ 1 and f (y) = 1 are true.

⇒ f –1 (1) = y

Q.7. Let R be the set of real numbers and f : R → R be such that for all x and y in R |f (x) – f(y) | ≤ | x – y |^3 . Prove that f(x) is a constant

Solution.

Since $|f(x)-f(y)| \leq |x-y|^3$ is true $\forall\ x, y \in R$

We have for $x \neq y$, $\dfrac{|f(x)-f(y)|}{|x-y|} \leq |x-y|^2$

$$\Rightarrow \lim_{y \to x}\left|\frac{f(x)-f(y)}{x-y}\right| \leq \lim_{y \to x}|x-y|^2$$

$\Rightarrow\ |f'(x)| \leq 0 \ \Rightarrow f'(x) = 0$

$\Rightarrow\ f(x)$ is a constant function. Hence Proved.

Q.8. Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function 'f' satisfies the relation f(x + y) = f (x) f (y) for all natural numbers x, y and further f(1) = 2.

Solution. Given that $f(x+y) = f(x)f(y)\ \forall\ x, y \in N$ and $f(1) = 2$

To find 'a' such that,

$$\sum_{k=1}^{n} f(a+k) = 16(2^n - 1) \quad ...(1)$$

For this we start with ...(2)

f (1) = 2

Then f (2) = f (1 + 1) = f (1) f (1)

⇒ f (2) = 2^2

Similarly we get, $f(3) = 2^3$,

$f(4) = 2^4, ..., f(n) = 2^n$

Now eq. (1) can be written as

$f(a+1) + f(a+2) + f(a+3) + ... + f(a+n) = 16(2^n - 1)$

$\Rightarrow\ f(a)f(1) + f(a)f(2) + f(a)f(3) + ... + f(a)f(n) = 16(2^n - 1)$

$\Rightarrow\ f(a)f(1) + f(a)f(2) + f(a)f(3) + ... + f(a)f(n) = 16(2^n - 1)$

$\Rightarrow\ f(a)[f(1) + f(2) + f(3) + ... + f(n)] = 16[2^n - 1]$

$\Rightarrow\ f(a)[2 + 2^2 + 2^3 + ... + 2^n] = 16[2^n - 1]$

$$\Rightarrow\ f(a)\left[\frac{2(2^n - 1)}{2 - 1}\right] = 16[2^n - 1]$$

$\Rightarrow\ f(a) = 8 = 2^3 = f(3) \Rightarrow a = 3$

Q.9. Let {x} and [x] denotes the fractional and integral part of a real number x respectively. Solve 4{x} = x + [x].

Solution. Given that 4 {x} = x + [x]

Where [x] = greatest integer ≤ x

$\{x\}$ = fractional part of x

$\therefore x = [x] + \{x\}$ for any $x \in R$

$\therefore$ Given eq^n becomes

$4\{x\} = [x] + \{x\} + [x] \Rightarrow 3\{x\} = 2[x]$

$\Rightarrow [x] = \frac{3}{2}\{x\}$ ....(1)

Now $-1 < \{x\} < 1$

$\Rightarrow -\frac{3}{2} < \frac{3}{2}\{x\} < \frac{3}{2}$

$\Rightarrow -\frac{3}{2} < [x] < \frac{3}{2}$ [Using eq^n (1)]

$\Rightarrow [x] = -1, 0, 1$

If $[x] = -1$

$\Rightarrow -1 = \frac{3}{2}\{x\}$ [Using eq^n (1)]

$\Rightarrow \{x\} = -\frac{2}{3}$

$\therefore x = [x] + \{x\}$

$\Rightarrow x = -1 + (-2/3) = -5/3$

If $[x] = 0$

$\Rightarrow \frac{3}{2}\{x\} = 0$ [Using eq^n (1)]

$\Rightarrow \{x\} = 0$

$\therefore x = 0 + 0 = 0$

If $[x] = 1$

$\Rightarrow \frac{3}{2}\{x\} = 1$ [Using eq^n (1)]

$\Rightarrow \{x\} = 2/3 \Rightarrow x = 1 + 2/3 = 5/3$

Thus, $x = -5/3, 0, 5/3$

Q.10. A function f :IR → IR, where IR is the set of real numbers, is defined by $f(x)=\dfrac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$. **Find the interval of values of α for which f is onto. Is the function one-to-one for α = 3?**

Solution.

Let us put $y=\dfrac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$

$\Rightarrow\ (\alpha+6x-8x^2)\,y=\alpha x^2+6x-8$

$\Rightarrow\ (\alpha+8y)\,x^2+6\,(1-y)\,x-(8+\alpha y)=0$

Since x is real, $D\geq 0$

$\Rightarrow\ 36\,(1-y)^2+4\,(\alpha+8y)\,(8+\alpha y)\geq 0$

$\Rightarrow\ 9\,(1-2y+y^2)+[8\alpha+(64+\alpha^2)y+8\alpha y^2]\geq 0$

$\Rightarrow\ y^2\,(9+8\alpha)+y\,(46+\alpha^2)+(9+8\alpha)\geq 0$ ...(1)

For (1) to hold for each $y\in R$, $9+8\alpha>0$

and $(46+\alpha^2)^2-4\,(9+8\alpha)^2\leq 0 \Rightarrow \alpha>-9/8$

and $[46+\alpha^2-2\,(9+8\alpha)]\,[46+\alpha^2+2\,(9+8\alpha)]\leq 0$

$\Rightarrow\ \alpha>-9/8$

and $(\alpha^2-16\alpha+28)\,(\alpha^2+16\alpha+64)\leq 0 \Rightarrow\ \alpha>-9/8$

and $(\alpha-2)\,(\alpha-14)\,(\alpha+8)^2\leq 0 \Rightarrow \alpha>-8/9$

and $(\alpha-2)\,(\alpha-14)\leq 0$ $\quad[\because (\alpha+8)^2\geq 0]$

$\Rightarrow \alpha>-8/9$ and $2\leq\alpha\leq 14\ \Rightarrow\ 2\leq\alpha\leq 14$

Thus, $f(x)=\dfrac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$ will be onto if $2\leq\alpha\leq 14$.

When $\alpha=3$

$f(x)=\dfrac{3x^2+6x-8}{3+6x-8x^2}$

In this case $f(x)=0$ implies, $3x^2+6x-8=0$

$\Rightarrow\ x=\dfrac{-6\pm\sqrt{36+96}}{6}=\dfrac{-6\pm\sqrt{132}}{6}=\dfrac{-6\pm 2\sqrt{33}}{6}$

$=\dfrac{1}{3}(-3\pm\sqrt{33})$

This shows that

$f\left[\dfrac{1}{3}(-3+\sqrt{33})\right]=f\left[\dfrac{1}{3}(-3-\sqrt{33})\right]=0$

Therefore, f is not one-to-one at $\alpha = 3$.

Q.11. Let $f(x) = Ax^2 + Bx + C$ where A, B, C are real numbers. Prove that if f(x) is an integer whenever x is an integer, then the numbers 2A, A + B and C are all integers. Conversely, prove that if the numbers 2A, A+B and C are all integers then f(x) is an integer whenever x is an integer.

Solution. Suppose $f(x) = Ax^2 + Bx + C$ is an integer whenever x is an integer.

$\therefore$ f (0), f (1), f (–1) are integers

$\Rightarrow$ C, A + B + C, A – B + C are integers.

$\Rightarrow$ C, A + B, A – B are integers

$\Rightarrow$ C, A + B, (A + B) + (A – B) = 2A are integers.

Conversely suppose 2A, A + B and C are integers.

Let x be any integer.

We have

$f(x) = Ax^2 + Bx + C$

$$= 2A\left[\frac{x(x-1)}{2}\right] + (A+B)\,x + C$$

Since x is an integer x, x (x – 1)/2 is an integer.

Also 2A, A + B and C are integers.

We get f (x) is an integer for all integer x.

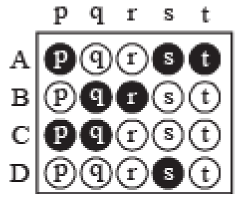
Match the following Question

Match the following Question

Direction (Q. 1 and 2) each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D,

while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column II.

The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q. 1. Let the function defined in column 1 have domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and range (-∞,∞)

Column I		Column II	
		(p)	onto but not one-one
(A)	$1+2x$	(q)	one- one but not onto
(B)	$\tan x$	(r)	one- one and onto
		(s)	neither one-one nor onto

Ans. (A) - q; (B) – r

Solution. (A) f (x) = 1 + 2x, Df = (–π/2, π/2)

The given function represents a straight line so it is one one.

But $f_{\min} = 1 - \pi = f\left(-\frac{\pi}{2}\right)$, $f_{\max} = 1 + \pi = f\left(\frac{\pi}{2}\right)$

$\therefore$ Range $f = (1-\pi, 1+\pi) \in (-\infty, \infty)$

$\therefore f$ is not onto. Hence (A) $\to$ (q).

(B) f (x) = tan x It is an increasing function on $(-\pi/2, \pi/2)$ and its range $= (-\infty,\infty)$ = co-domain of f.

∴ f is one one onto.

∴ (B) → r

Q. 2. Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I		*Column II*
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, then $f(x)$ satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	(s)	$f(x) < 1$

Ans. (A) -p, r, s; (B) - q, s ; (C) - q, s ; (D) -p, r, s

Solution.

We have $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6} = \frac{(x-5)(x-1)}{(x-2)(x-3)}$

Integer Type question

Q. 1. Let f : [0, 4π] → [0, π] be defined by f (x) = $\cos^{-1}$(cos x). The number of points x ∈[0,4π] satisfying the equation

$f(x) = \frac{10 - x}{10}$ is

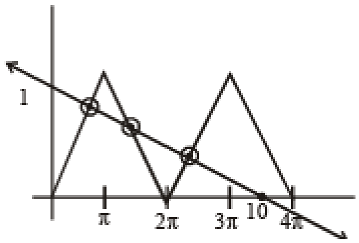
Ans. 3

Solution.

We have $f:[0,4\pi]\to[0,\pi]$

$f(x)=\cos^{-1}(\cos x)$

and $g(x)=\frac{10-x}{10}=1-\frac{x}{10}$



The graph of y = f (x) and y = g (x) are as follows.

Clearly f(x) = g(x) has 3 solutions.